附录 1

引理 1 如果$Y \sim AEPD\left(\mu,\sigma,\tau,p_1,p_2\right)$，则$Y$的$\tau$分位数恰好是它的位置参数$\mu$，即，$Q_\tau\left(Y\right)=\mu$.

证明：要证明引理 1 成立就是要证明$P\left(Y\le\mu\right)=\tau$.接下来计算基于 AEP 分布的累积分布函数在$y=\mu$处的值：

$$P\left(Y\le\mu\right)=\int_{-\infty}^{\mu}\frac{1}{\sigma}\exp\left(-\left|\frac{y-\mu}{\tau\sigma/\Gamma(1+1/p_1)}\right|^{p_1}\right)I\left(y\le\mu\right)dy$$

* MERGEFORMAT (1)

不失一般性地假设μ=0，σ=1，则式化简为

$$P\left(Y\le\mu\right)=\int_{-\infty}^{0}\exp\left(-\left(\frac{-y}{\tau/\Gamma(1+1/p_1)}\right)^{p_1}\right)dy$$

* MERGEFORMAT (2)

令$x=\left(-y\right)^{p_1}$，把式中的y用x替换可得

$$P\left(Y\le\mu\right)=\int_{0}^{+\infty}\exp\left(-\left(\frac{1}{\tau/\Gamma(1+1/p_1)}\right)^{p_1}x\right)\frac{1}{p_1}x^{1/p_1-1}dx$$

* MERGEFORMAT (3)

式中的被积表达式经过整理可知是形状参数为$1/p_1$,尺度参数为$\left(\frac{1}{\tau/\Gamma(1+1/p_1)}\right)^{p_1}$的伽马分布的核函数，因此对式求积分可得

$$P\left(Y\le\mu\right)=\frac{1}{p_1}\Gamma(\frac{1}{p_1})\left(\left(\frac{\tau}{\Gamma(1+1/p_1)}\right)^{p_1}\right)^{1/p_1}$$

* MERGEFORMAT (4)

再通过伽马函数的性质$\Gamma\left(x+1\right)=x\Gamma\left(x\right)$可得$P\left(Y\le\mu\right)=\tau$。

附录2

引理2 基于AEP分布的似然函数最大化可推导出损失函数最小化成立，即$\sum_{i=1}^{N}\sum_{t=1}^{T}\rho_\tau(\mu)=\sum_{i=1}^{N}\sum_{t=1}^{T}\left\{(1\text{-}\tau)|\mu|I(\mu\le 0)+\tau|\mu|I(\mu>0)\right\}$取最小，其中$\mu=y_{it}-x_{it}^T\beta-\alpha_i$。

证明：面板数据模型基于AEP分布的似然函数如式所示

$$
\begin{aligned}
L(\beta,\sigma,p_1,p_2|Y,X)=\prod_{i=1}^{N}\prod_{t=1}^{T}&\left\{\frac{1}{\sigma}\exp\left[-\left(\frac{x_{it}^T\beta+\alpha_i-y_{it}}{\tau\sigma/\Gamma(1+1/p_1)}\right)^{p_1}\right]I\left(y_{it}\le x_{it}^T\beta+\alpha_i\right)\right.\\
&\left.+\frac{1}{\sigma}\exp\left[-\left(\frac{y_{it}-x_{it}^T\beta-\alpha_i}{(1-\tau)\sigma/\Gamma(1+1/p_2)}\right)^{p_2}\right]I\left(y_{it}>x_{it}^T\beta+\alpha_i\right)\right\}\\
&=\frac{1}{\sigma^{NT}}\exp\left[-\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{x_{it}^T\beta+\alpha_i-y_{it}}{\tau\sigma/\Gamma(1+1/p_1)}\right)^{p_1}\right]I\left(y_{it}\le x_{it}^T\beta+\alpha_i\right)\\
&+\frac{1}{\sigma^{NT}}\exp\left[-\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{y_{it}-x_{it}^T\beta-\alpha_i}{(1-\tau)\sigma/\Gamma(1+1/p_2)}\right)^{p_2}\right]I\left(y_{it}>x_{it}^T\beta+\alpha_i\right)
\end{aligned}
$$

* MERGEFORMAT (5)

把σ看作冗余参数，记$\mu=y_{it}-x_{it}^T\beta-\alpha_i$，则最大化似然函数式等价于最小化式

$$
\sum_{i=1}^{N}\sum_{t=1}^{T}\left\{\left(\frac{|\mu|}{\tau\sigma/\Gamma(1+1/p_1)}\right)^{p_1}I(\mu\le 0)+\left(\frac{|\mu|}{(1-\tau)\sigma/\Gamma(1+1/p_2)}\right)^{p_2}I(\mu>0)\right\}
$$

* MERGEFORMAT (6)

Yu和Moyeed （2001）提出，基于ALD的似然函数最大化等价于损失函数最小化，而ALD是AEP分布尾部参数p_1=p_2=1时的一个特例，因此，令$p_1=p_2$=1，由$\Gamma(2)$=1，把式化简可得式

$$
\sum_{i=1}^{N}\sum_{t=1}^{T}\left\{\frac{1}{\tau\sigma}|\mu|I(\mu\le 0)+\frac{1}{(1-\tau)\sigma}|\mu|I(\mu>0)\right\}
$$

* MERGEFORMAT (7)

参考Komunjer（2007）以及Zhu和Zinde-Walsh（2009）等重新参数化的过程，令$\sigma=\frac{1}{\tau(1\text{-}\tau)}$，则式变形为式

$$
\sum_{i=1}^{N}\sum_{t=1}^{T}\left\{(1-\tau)|\mu|I(\mu\le 0)+\tau|\mu|I(\mu>0)\right\}
$$

* MERGEFORMAT (8)

式即为损失函数$\sum_{i=1}^{N}\sum_{t=1}^{T}\rho_\tau(\mu)$的表达式形式，由此可见基于AEP分布的似然函数最大化是损失函数最小化的充分条件，基于AEP分布的似然函数最大化成立必然有损失函数最小化成立。

附表 1

表 1　　三种方法的自适应 Lasso 功能比较

误差分布	参数	ALD ($T=10,N=30$)			SEP ($T=10,N=30$)			AEP ($T=10,N=30$)		
		$\tau=0.1$	$\tau=0.5$	$\tau=0.9$	$\tau=0.1$	$\tau=0.5$	$\tau=0.9$	$\tau=0.1$	$\tau=0.5$	$\tau=0.9$
正态分布	MAD	1.546	1.442	1.468※	1.674	1.563	1.732	1.411※	1.437※	1.656
	β_1	1.090	1.080	0.981※	0.960	1.089	1.088	0.996※	1.040※	1.113
		(0.069)	(0.082)	(0.083)	(0.237)	(0.242)	(0.195)	(0.055)※	(0.069)※	(0.049)※
		[0.954, 1.226]	[0.928, 1.252]	[0.818, 1.143]	[0.495, 1.425]	[0.614, 1.564]	[0.704, 1.472]	[0.888, 1.104]	[0.905, 1.175]	[1.016, 1.210]
	β_2	-0.076	-0.075	-0.078※	-0.125	-0.209	-0.324	-0.050※	-0.057※	-0.090
		(0.090)	(0.104)	(0.118)※	(0.280)	(0.225)	(0.281)	(0.067)※	(0.090)※	(0.135)
		[-0.252, 0.099]	[-0.280, 0.129]	[-0.311, 0.154]	[-0.673, 0.423]	[-0.651, 0.232]	[-0.875, 0.227]	[-0.180, 0.080]	[-0.232, 0.117]	[-0.356, 0.174]
	β_3	0.103	0.085※	0.086	0.171	0.227	0.447	-0.033※	0.142	0.073※
		(0.135)	(0.109)	(0.133)	(0.245)	(0.258)	(0.258)	(0.063)※	(0.108)※	(0.115)※
		[-0.162, 0.368]	[-0.128, 0.299]	[-0.176, 0.348]	[-0.309, 0.652]	[-0.277, 0.733]	[-0.058, 0.953]	[-0.155, 0.090]	[-0.0689, 0.353]	[-0.153, 0.300]
	β_4	2.697	2.718	2.690※	2.441	2.349	2.044	2.915※	2.751※	2.561
		(0.115)	(0.110)	(0.126)	(0.255)	(0.250)	(0.271)	(0.064)※	(0.082)※	(0.095)※
		[2.472, 2.923]	[2.502, 2.933]	[2.442, 2.938]	[1.941, 2.940]	[1.859, 2.839]	[1.511, 2.576]	[2.791, 3.040]	[2.590, 2.912]	[2.373, 2.748]
	β_5	0.102	0.096	0.117	0.144	0.208	0.300	0.001※	0.073※	0.081※
		(0.097)	(0.095)	(0.125)	(0.222)	(0.222)	(0.214)	(0.081)※	(0.082)※	(0.110)※
		[-0.089, 0.292]	[-0.090, 0.283]	[-0.128, 0.362]	[-0.292, 0.579]	[-0.227, 0.643]	[-0.119, 0.719]	[-0.158, 0.159]	[-0.0882, 0.234]	[-0.136, 0.299]
	β_6	0.045	0.057	0.050※	0.036	0.109	0.194	0.027※	0.047※	0.066
		(0.069)	(0.071)※	(0.075)	(0.242)	(0.195)	(0.186)	(0.069)※	(0.097)	(0.072)※
		[-0.091, 0.180]	[-0.081, 0.197]	[-0.097, 0.197]	[-0.438, 0.511]	[-0.274, 0.492]	[-0.169, 0.559]	[-0.108, 0.162]	[-0.142, 0.238]	[-0.074, 0.208]
	β_7	0.010※	-0.012	0.009	-0.070	0.027	0.119	-0.077	-0.001※	0.008※
		(0.055)※	(0.046)	(0.052)※	(0.313)	(0.224)	(0.254)	(0.080)	(0.045)※	(0.085)
		[-0.098, 0.119]	[-0.103, 0.078]	[-0.094, 0.112]	[-0.683, 0.544]	[-0.412, 0.466]	[-0.379, 0.618]	[-0.234, 0.081]	[-0.090, 0.087]	[-0.158, 0.176]
	β_8	-0.001※	0.004※	-0.003※	0.053	0.033	0.108	0.003	-0.010	-0.106
		(0.044)	(0.044)	(0.054)	(0.321)	(0.240)	(0.238)	(0.013)※	(0.039)※	(0.046)※
		[-0.090, 0.087]	[-0.082, 0.091]	[-0.110, 0.103]	[-0.576, 0.681]	[-0.437, 0.505]	[-0.359, 0.576]	[-0.023, 0.029]	[-0.087, 0.066]	[-0.198, -0.014]
t 分布	MAD	1.879	1.561	1.867	1.886	1.622	1.678	1.741※	1.475※	1.637※
	β_1	0.716	0.839	0.670	0.724	1.038※	0.889	0.771※	0.889	1.058※
		(0.405)※	(0.452)	(0.418)	(0.651)	(0.259)	(0.436)	(0.400)	(0.231)※	(0.242)※
		[-0.079, 1.511]	[-0.047, 1.727]	[-0.150, 1.491]	[-0.551, 1.999]	[0.529, 1.546]	[0.034, 1.744]	[-0.013, 1.555]	[0.436, 1.343]	[0.581, 1.534]
	β_2	0.091	0.137	0.065	0.040	-0.132	0.095	0.031※	-0.012※	-0.031※
		(0.330)	(0.335)	(0.430)	(0.661)	(0.404)	(0.613)	(0.527)※	(0.205)※	(0.338)※
		[-0.555, 0.737]	[-0.519, 0.794]	[-0.779, 0.909]	[-1.256, 1.337]	[-0.924, 0.660]	[-1.104, 1.295]	[-1.001, 1.064]	[-0.415, 0.391]	[-0.695, 0.632]
	β_3	0.052※	0.081	0.031※	0.325	-0.054	-0.209	0.185	0.038※	-0.265

		(0.175)※ [-0.291, 0.396]	(0.258) [-0.424, 0.586]	(0.162) [-0.287, 0.349]	(0.561) [-0.774, 1.425]	(0.232) [-0.510, 0.400]	(0.411) [-1.015, 0.597]	(0.168) [-0.145, 0.515]	(0.188)※ [-0.331, 0.408]	(0.119)※ [-0.500, -0.030]
	β_4	2.841 (0.291) [2.270, 3.413]	2.843 (0.245) [2.361, 3.325]	2.753 (0.299) [2.166, 3.340]	2.713 (0.588)※ [1.558, 3.869]	3.120 (0.215) [2.697, 3.543]	3.113 (0.358) [2.408, 3.817]	2.850※ (0.287) [2.286, 3.414]	3.017※ (0.188)※ [2.648, 3.387]	3.100※ (0.211)※ [2.686, 3.515]
	β_5	0.015※ (0.137)※ [-0.253, 0.285]	0.007※ (0.099) [-0.188, 0.203]	0.086 (0.220)※ [-0.345, 0.517]	0.018 (0.559) [-1.078, 1.114]	0.027 (0.271) [-0.505, 0.561]	0.066 (0.332) [-0.584, 0.717]	0.048 (0.073) [-0.095, 0.193]	-0.057 (0.084)※ [-0.224, 0.108]	-0.051※ (0.252) [-0.547, 0.444]
	β_6	-0.190 (0.249) [-0.679, 0.299]	-0.207 (0.245) [-0.688, 0.274]	-0.282 (0.292)※ [-0.855, 0.290]	-0.276 (0.665) [-1.582, 1.030]	-0.440 (0.357) [-1.140, 0.260]	-0.401 (0.339) [-1.066, 0.263]	-0.001※ (0.573)※ [-1.123, 1.120]	-0.100※ (0.203)※ [-0.500, 0.299]	-0.202※ (0.315) [-0.820, 0.415]
	β_7	0.062 (0.155) [-0.242, 0.367]	0.078 (0.125)※ [-0.167, 0.324]	0.108 (0.230) [-0.343, 0.560]	0.204 (0.587) [-0.947, 1.356]	0.443 (0.320) [-0.184, 1.071]	0.337 (0.389) [-0.426, 1.101]	0.012※ (0.055)※ [-0.096, 0.122]	0.025※ (0.229) [-0.424, 0.476]	0.008※ (0.162)※ [-0.309, 0.326]
	β_8	0.029 (0.104) [-0.175, 0.234]	0.047 (0.119) [-0.186, 0.282]	0.023 (0.082) [-0.138, 0.186]	0.004※ (0.381)※ [-0.742, 0.750]	0.017 (0.259) [-0.492, 0.526]	0.076 (0.440) [-0.787, 0.939]	0.017 (0.087) [-0.153, 0.187]	0.002※ (0.062)※ [-0.121, 0.125]	0.014※ (0.051)※ [-0.086, 0.116]
	MAD	1.842	1.797	1.871	1.789	1.812	1.893	1.767※	1.784※	1.656※
	β_1	0.624 (0.322) [-0.008, 1.257]	0.655 (0.331)※ [0.007, 1.304]	0.543 (0.339) [-0.123, 1.209]	0.694 (0.425) [-0.138, 1.527]	0.637 (0.429) [-0.204, 1.478]	0.677 (0.421) [-0.149, 1.503]	0.741※ (0.234)※ [0.281, 1.201]	0.737※ (0.348) [0.054, 1.420]	0.804※ (0.319)※ [0.178, 1.430]
	β_2	-0.075 (0.193)※ [-0.454, 0.303]	-0.064 (0.239) [-0.534, 0.405]	-0.062※ (0.324) [-0.695, 0.570]	0.009※ (0.360) [-0.696, 0.715]	-0.037 (0.319) [-0.664, 0.589]	-0.104 (0.350) [-0.791, 0.583]	0.011 (0.207) [-0.395, 0.418]	-0.029※ (0.211)※ [-0.443, 0.384]	-0.263 (0.317)※ [-0.885, 0.358]
	β_3	0.216 (0.264) [-0.301, 0.734]	0.259 (0.296) [-0.320, 0.839]	0.255 (0.285)※ [-0.303, 0.815]	0.330 (0.441) [-0.534, 1.195]	0.284 (0.351) [-0.404, 0.973]	0.357 (0.394) [-0.414, 1.129]	0.129※ (0.252)※ [-0.364, 0.624]	0.248※ (0.284)※ [-0.308, 0.805]	0.226※ (0.359) [-0.476, 0.930]
卡方分布	β_4	2.649 (0.568) [1.536, 3.763]	2.628 (0.453) [1.739, 3.517]	2.689 (0.393) [1.918, 3.461]	2.509 (0.590) [1.350, 3.668]	2.602 (0.588) [1.448, 3.756]	2.413 (0.628) [1.181, 3.645]	2.653※ (0.485)※ [1.700, 3.605]	2.656※ (0.433)※ [1.805, 3.506]	2.706※ (0.372)※ [1.976, 3.436]
	β_5	0.142 (0.266) [-0.380, 0.665]	0.123 (0.250) [-0.366, 0.613]	0.118 (0.249) [-0.370, 0.607]	0.159 (0.363) [-0.551, 0.871]	0.103※ (0.269) [-0.425, 0.631]	0.185 (0.359) [-0.520, 0.890]	0.136※ (0.260)※ [-0.373, 0.646]	0.119 (0.222)※ [-0.316, 0.554]	0.117※ (0.220)※ [-0.314, 0.550]
	β_6	-0.143 (0.204) [-0.544, 0.257]	-0.128 (0.287) [-0.692, 0.434]	-0.193 (0.304) [-0.791, 0.403]	-0.145 (0.370) [-0.870, 0.579]	-0.128 (0.386) [-0.884, 0.628]	0.044 (0.391) [-0.721, 0.810]	-0.061※ (0.131)※ [-0.319, 0.196]	-0.106※ (0.194)※ [-0.487, 0.275]	0.014※ (0.250)※ [-0.476, 0.505]
	β_7	0.019 (0.263) [-0.496, 0.535]	-0.026※ (0.248) [-0.513, 0.461]	0.062 (0.233) [-0.394, 0.520]	0.038 (0.385) [-0.717, 0.795]	0.084 (0.371) [-0.643, 0.812]	0.001※ (0.466) [-0.914, 0.916]	0.011※ (0.236)※ [-0.452, 0.476]	-0.075 (0.208)※ [-0.484, 0.334]	-0.025 (0.218)※ [-0.452, 0.401]

	β_8	-0.062※ (0.218)※ [-0.490, 0.366]	-0.027 (0.165) [-0.352, 0.296]	-0.123 (0.196)※ [-0.508, 0.261]	-0.071 (0.293) [-0.646, 0.504]	-0.063 (0.295) [-0.642, 0.516]	0.031 (0.267) [-0.492, 0.555]	-0.077 (0.229) [-0.526, 0.372]	-0.018※ (0.121)※ [-0.257, 0.219]	0.009※ (0.241) [-0.463, 0.483]
误差分布	参数	ALD （$T=50$，$N=50$）			SEP （$T=50$，$N=50$）			AEP （$T=50$，$N=50$）		
		$\tau=0.1$	$\tau=0.5$	$\tau=0.9$	$\tau=0.1$	$\tau=0.5$	$\tau=0.9$	$\tau=0.1$	$\tau=0.5$	$\tau=0.9$
正态分布	MAD	1.113	0.952	1.235	0.976	0.947	1.048	0.964※	0.931※	1.027※
	β_1	1.026 (0.032) [0.963, 1.088]	1.009 (0.175) [0.665, 1.352]	1.042※ (0.021) ※ [1.001, 1.083]	1.017※ (0.027) [0.964, 1.069]	1.011 (0.078) [0.858, 1.163]	1.076 (0.143) [0.796, 1.355]	1.023 (0.025) ※ [0.973, 1.072]	1.008※ (0.053) ※ [0.904, 1.112]	1.051 (0.028) [0.996, 1.105]
	β_2	0.021 (0.116) [-0.206, 0.2482]	0.006 (0.171) [-0.328, 0.340]	0.019 (0.167) [-0.308, 0.3466]	0.015※ (0.093) [-0.167, 0.197]	0.008 (0.185) [-0.354, 0.370]	0.023 (0.087) [-0.147, 0.193]	0.017 (0.069) ※ [-0.118, 0.152]	0.006※ (0.117)※ [-0.223, 0.235]	-0.014※ (0.041) ※ [-0.094, 0.066]
	β_3	0.012 (0.174) [-0.329, 0.353]	0.003※ (0.141) [-0.272, 0.278]	0.008 (0.214) [-0.411, 0.427]	0.014 (0.135) [-0.250, 0.278]	0.007 (0.269) [-0.520, 0.534]	0.016 (0.046)※ [-0.074, 0.106]	0.009※ (0.113)※ [-0.212, 0.230]	0.004 (0.098)※ [-0.187, 0.195]	0.007※ (0.128) [-0.243, 0.257]
	β_4	2.935 (0.042) [2.852, 3.017]	2.976 (0.275) [2.436, 3.515]	2.927 (0.104) [2.723, 3.130]	2.942 (0.071) [2.802, 3.081]	2.982※ (0.186) [2.616, 3.347]	2.938 (0.191) [2.564, 3.311]	2.956※ (0.036)※ [2.885, 3.026]	2.979 (0.154)※ [2.676, 3.281]	2.943※ (0.050)※ [2.845, 3.041]
	β_5	-0.026 (0.166) [-0.351, 0.299]	0.015 (0.113) [-0.206, 0.236]	0.024 (0.164) [-0.297, 0.345]	0.028 (0.153) [-0.272, 0.328]	0.015 (0.196) [-0.369, 0.399]	0.018※ (0.159) [-0.293, 0.329]	0.019※ (0.026)※ [-0.032, 0.070]	0.013※ (0.102)※ [-0.186, 0.212]	-0.021 (0.120)※ [-0.256, 0.214]
	β_6	0.011※ (0.168) [-0.318, 0.340]	0.009 (0.153) [-0.290, 0.308]	0.016 (0.193) [-0.361, 0.393]	0.012 (0.090) ※ [-0.164, 0.188]	0.010 (0.148) [-0.280, 0.300]	0.013 (0.128) [-0.237, 0.263]	-0.014 (0.185) [-0.376, 0.348]	0.008※ (0.102)※ [-0.192, 0.208]	0.009※ (0.072)※ [-0.132, 0.150]
	β_7	0.008 (0.123) [-0.232, 0.248]	0.007 (0.223) [-0.430, 0.444]	0.008 (0.124) [-0.235, 0.251]	0.008 (0.120) [-0.227, 0.243]	0.006 (0.139) [-0.266, 0.278]	0.009 (0.151) [-0.284, 0.302]	0.005※ (0.107) ※ [-0.204, 0.214]	0.004※ (0.121)※ [-0.232, 0.240]	0.003※ (0.087)※ [-0.167, 0.173]
	β_8	0.005 (0.123) [-0.235, 0.245]	0.004 (0.231) [-0.448, 0.456]	0.008 (0.139) [-0.264, 0.280]	0.007 (0.225) [-0.434, 0.448]	0.005 (0.129) ※ [-0.247, 0.257]	-0.007 (0.121) [-0.244, 0.230]	0.002※ (0.114)※ [-0.221, 0.225]	-0.001※ (0.248) [-0.487, 0.485]	-0.004※ (0.116)※ [-0.231, 0.223]
t分布	MAD	1.378	1.238	1.473	1.413	1.205	1.426	1.347※	1.126※	1.401※
	β_1	0.882 (0.104) [0.678, 1.085]	0.936 (0.091) [0.757, 1.114]	0.856 (0.074) [0.710, 1.001]	0.875 (0.051)※ [0.775, 0.974]	0.941 (0.159) [0.629, 1.252]	0.869 (0.240) [0.399, 1.338]	0.886※ (0.067) [0.754, 1.017]	0.947※ (0.082)※ [0.786, 1.107]	0.872※ (0.051)※ [0.772, 0.971]
	β_2	0.017※ (0.165) [-0.306, 0.340]	0.025 (0.095)※ [-0.160, 0.210]	0.041 (0.213) [-0.376, 0.458]	0.031 (0.187) [-0.335, 0.397]	0.021 (0.107) [-0.188, 0.230]	0.036 (0.151) [-0.260, 0.332]	0.023 (0.046)※ [-0.067, 0.113]	0.018※ (0.149) [-0.274, 0.310]	0.027※ (0.142)※ [-0.251, 0.305]
	β_3	0.022 (0.174)	0.016 (0.143)	0.009※ (0.118)	0.019 (0.165)	0.014 (0.117)	0.012 (0.139)	0.018※ (0.123)※	0.010※ (0.073)※	-0.013 (0.115) ※

		[-0.319, 0.363]	[-0.263, 0.295]	[-0.222, 0.240]	[-0.304, 0.342]	[-0.215, 0.243]	[-0.260, 0.284]	[-0.223, 0.259]	[-0.132, 0.152]	[-0.238, 0.212]
	β_4	3.107 (0.116) [2.879, 3.334]	3.093 (0.099) [2.899, 3.286]	3.101 (0.110) [2.885, 3.316]	3.112 (0.059) ※ [2.996, 3.227]	3.082 (0.071) ※ [2.943, 3.220]	3.087※ (0.092) [2.906, 3.267]	3.103※ (0.094) [2.918, 3.287]	3.051※ (0.087) [2.880, 3.221]	3.089 (0.056) ※ [2.979, 3.198]
	β_5	0.034 (0.203) [-0.364, 0.432]	0.019 (0.225) [-0.422, 0.460]	0.025※ (0.218) [-0.401, 0.451]	0.026 (0.151) ※ [-0.270, 0.322]	-0.020 (0.145) [-0.304, 0.264]	0.031 (0.216) [-0.392, 0.454]	0.015※ (0.210) [-0.397, 0.427]	-0.013※ (0.113)※ [-0.234, 0.208]	0.026 (0.132)※ [-0.232, 0.284]
	β_6	0.018 (0.134) [-0.244, 0.280]	-0.009 (0.135) [-0.273, 0.255]	0.016※ (0.167) [-0.311, 0.343]	-0.019 (0.127) [-0.267, 0.229]	-0.011 (0.204) [-0.410, 0.388]	0.023 (0.215) [-0.398, 0.444]	-0.016※ (0.098)※ [-0.208, 0.176]	-0.008※ (0.103)※ [-0.210, 0.194]	-0.019 (0.127)※ [-0.268, 0.230]
	β_7	-0.009 (0.135) [-0.273, 0.255]	0.007 (0.174) [-0.334, 0.348]	0.005※ (0.157) [-0.302, 0.312]	0.009 (0.112) ※ [-0.210, 0.228]	0.008 (0.146) [-0.278, 0.294]	-0.007 (0.131) [-0.264, 0.250]	-0.008※ (0.146) [-0.293, 0.277]	0.006※ (0.125)※ [-0.238, 0.250]	0.009 (0.106)※ [-0.199, 0.217]
	β_8	0.014 (0.231) [-0.438, 0.466]	-0.007 (0.141) [-0.282, 0.268]	0.011 (0.209) [-0.398, 0.420]	0.008 (0.186) [-0.356, 0.372]	-0.004 (0.117) ※ [-0.233, 0.225]	0.006※ (0.126) [-0.241, 0.253]	-0.006※ (0.134) ※ [-0.268, 0.256]	-0.002※ (0.154) [-0.303, 0.299]	0.008 (0.098) ※ [-0.183, 0.199]
	MAD	1.277	1.258	1.316	1.333	1.244	1.204	1.211※	1.203※	1.187※
	β_1	0.881 (0.084) [0.716, 1.045]	0.954 (0.095) [0.767, 1.140]	0.893 (0.094) [0.708, 1.077]	0.877 (0.117) [0.648, 1.105]	0.946 (0.144) [0.663, 1.228]	0.906 (0.137) [0.637, 1.174]	0.895※ (0.068)※ [0.761, 1.028]	0.965※ (0.076)※ [0.815, 1.114]	0.918※ (0.089)※ [0.743, 1.092]
	β_2	0.041 (0.095) [-0.145, 0.227]	0.018 (0.096) ※ [-0.170, 0.206]	-0.023 (0.062)※ [-0.144, 0.098]	0.028 (0.115) [-0.197, 0.253]	0.012※ (0.204) [-0.387, 0.411]	-0.016 (0.154) [-0.317, 0.285]	-0.025※ (0.080)※ [-0.182, 0.132]	0.014 (0.177) [-0.332, 0.360]	-0.011※ (0.135) [-0.275, 0.253]
	β_3	0.014 (0.146) [-0.272, 0.300]	0.015 (0.068)※ [-0.118, 0.148]	0.017 (0.153) [-0.283, 0.317]	0.020 (0.124) [-0.223, 0.263]	0.018 (0.167) [-0.308, 0.344]	0.027 (0.117) [-0.202, 0.256]	0.013※ (0.108)※ [-0.198, 0.224]	0.008※ (0.097) [-0.182, 0.198]	-0.016※ (0.066)※ [-0.145, 0.113]
卡方分布	β_4	3.102 (0.097) [2.911, 3.292]	2.946 (0.186) [2.580, 3.311]	2.938 (0.138) [2.667, 3.208]	3.097 (0.112) [2.877, 3.316]	2.952 (0.201) [2.557, 3.346]	2.965 (0.148) [2.674, 3.255]	3.061※ (0.097)※ [2.870, 3.251]	2.987※ (0.110)※ [2.771, 3.202]	2.972※ (0.127)※ [2.723, 3.220]
	β_5	-0.019 (0.165) [-0.343, 0.305]	0.014 (0.162) [-0.303, 0.331]	0.016 (0.123) [-0.224, 0.256]	0.017 (0.137) [-0.251, 0.285]	-0.009※ (0.142) [-0.288, 0.270]	0.021 (0.095) [-0.164, 0.206]	-0.006※ (0.085)※ [-0.172, 0.160]	-0.012 (0.136)※ [-0.279, 0.255]	0.009※ (0.079)※ [-0.145, 0.163]
	β_6	0.023 (0.196) [-0.360, 0.406]	0.018 (0.104) [-0.185, 0.221]	0.024 (0.116)※ [-0.203, 0.251]	0.027 (0.180) [-0.325, 0.379]	0.015 (0.125) [-0.230, 0.260]	0.019 (0.181) [-0.335, 0.373]	0.021※ (0.117)※ [-0.208, 0.250]	0.008※ (0.098)※ [-0.184, 0.200]	0.016※ (0.134) [-0.247, 0.279]
	β_7	0.007 (0.178) [-0.342, 0.356]	-0.004 (0.141) [-0.280, 0.272]	0.009 (0.146) [-0.277, 0.295]	0.009 (0.153) [-0.290, 0.308]	0.007 (0.169) [-0.324, 0.338]	0.006※ (0.157) [-0.301, 0.313]	0.005※ (0.126)※ [-0.242, 0.252]	0.003※ (0.132)※ [-0.255, 0.261]	0.008 (0.110)※ [-0.208, 0.224]
	β_8	0.009	0.007	0.008	0.006	0.006	0.008	-0.005※	0.004※	0.002※

		(0.125)	(0.135)	(0.123)	(0.117)	(0.142)	(0.121) ※	(0.105)※	(0.097) ※	(0.142)
		[-0.236, 0.254]	[-0.257, 0.271]	[-0.233, 0.249]	[-0.223, 0.235]	[-0.272, 0.284]	[-0.229, 0.245]	[-0.211, 0.201]	[-0.186, 0.194]	[-0.276, 0.280]

注：MAD 表示重复模拟 50 次的平均绝对误差的均值，用来反映整体的估计效果；回归参数的均值为重复模拟 50 次的参数估计值的平均值；小括号中的数值为标准差；方括号中的数值为 95%HPD 置信区间；相同分位数水平下，模拟中较好的估计结果用※标记。

附图 1

（一） 数据集出现极端值时 AEP 方法的参数取 2 个不同初值的双层迭代轨迹图

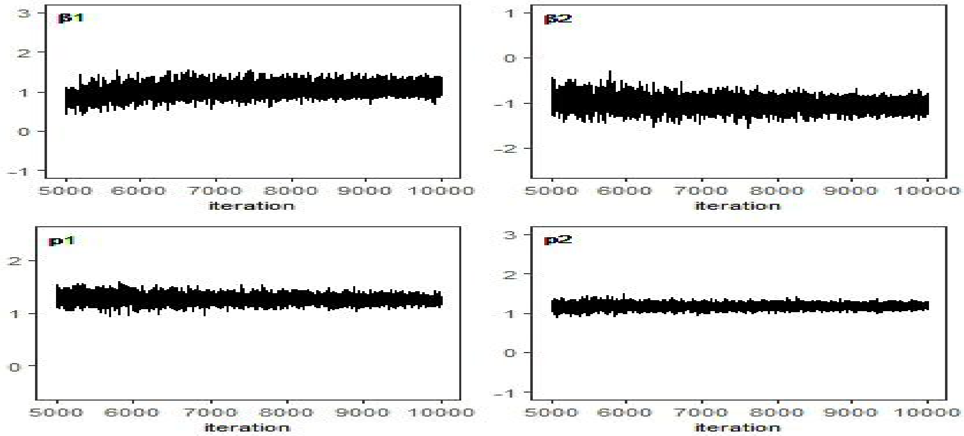

图 1 面板数据受极端值影响时，$\tau=0.1$ 分位点 AEP 方法的参数双层迭代轨迹图

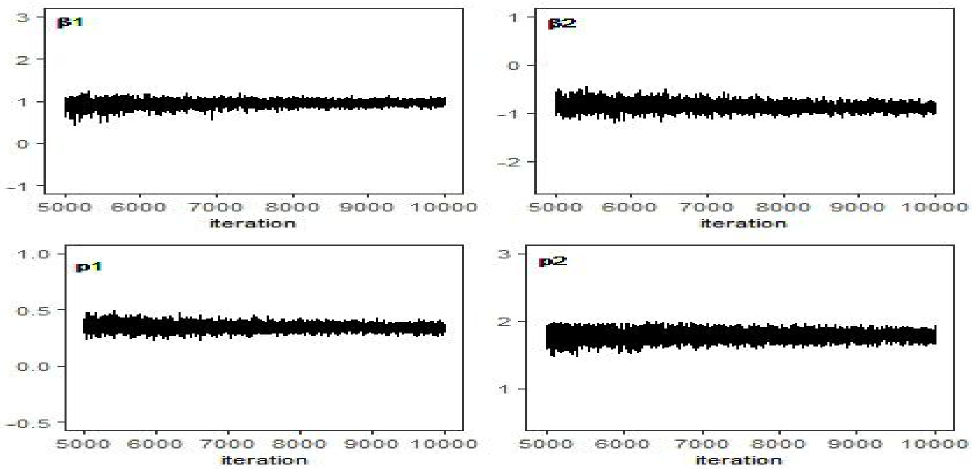

图 2 面板数据受极端值影响时，$\tau=0.5$ 分位点 AEP 方法的参数双层迭代轨迹图

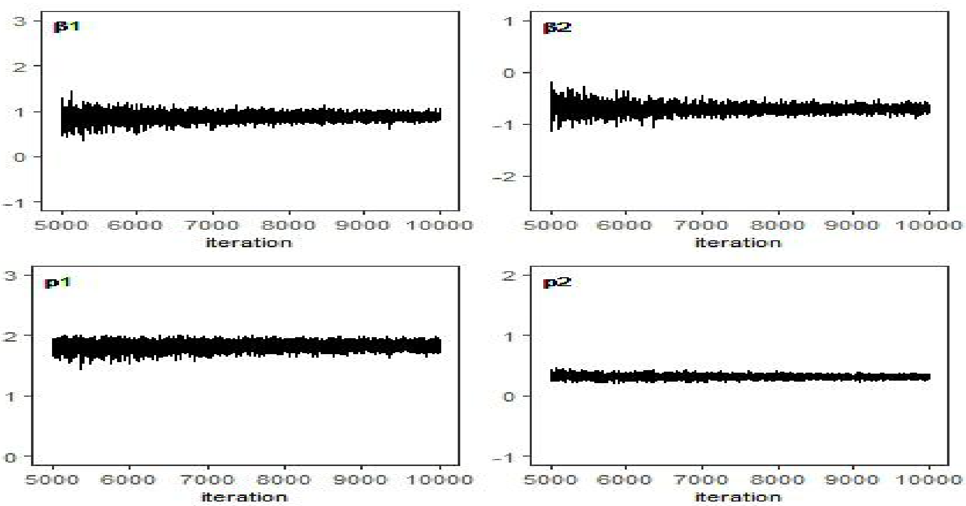

图 3 面板数据受极端值影响时，$\tau=0.9$ 分位点 AEP 方法的参数双层迭代轨迹图

附图2

（二）自适应Lasso功能模拟中AEP方法的参数取2个不同初值的双层迭代轨迹图

1.样本容量为$T=10$, $N=30$时，AEP方法的参数双层迭代轨迹图。

（1）误差为正态分布

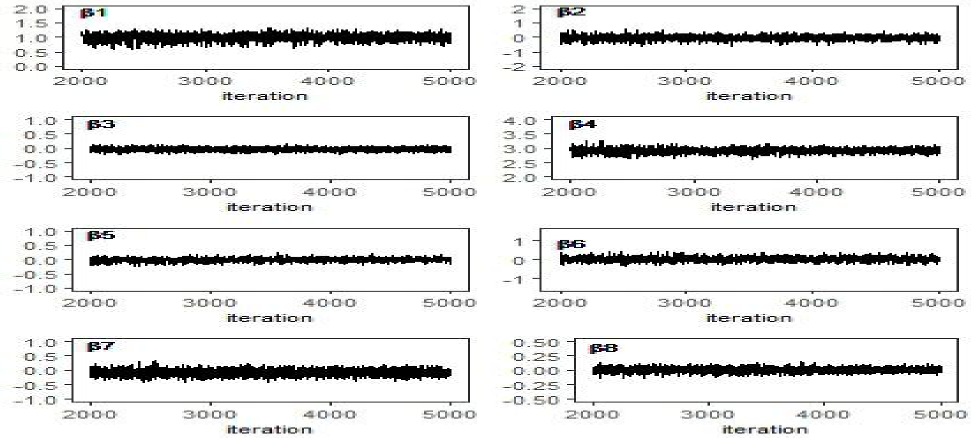

图1　误差为正态分布时，$T=10$，$N=30$，τ=0.1分位点AEP方法的参数双层迭代轨迹图

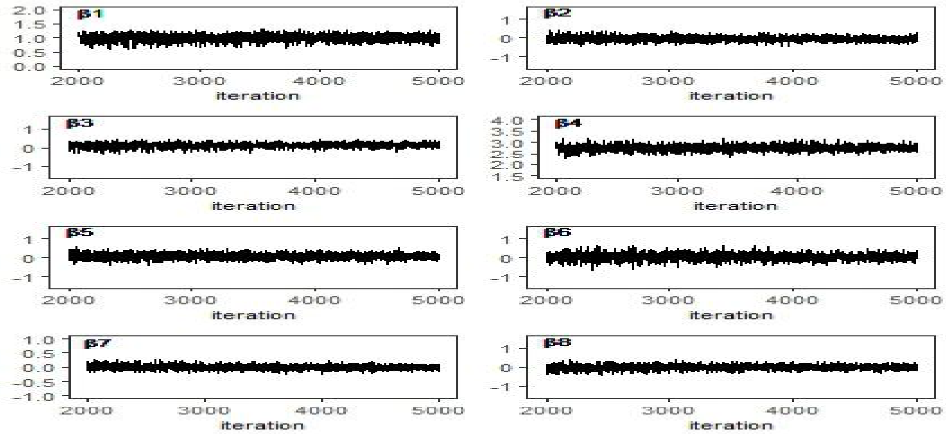

图2　误差为正态分布时，$T=10$，$N=30$，τ=0.5分位点AEP方法的参数双层迭代轨迹图

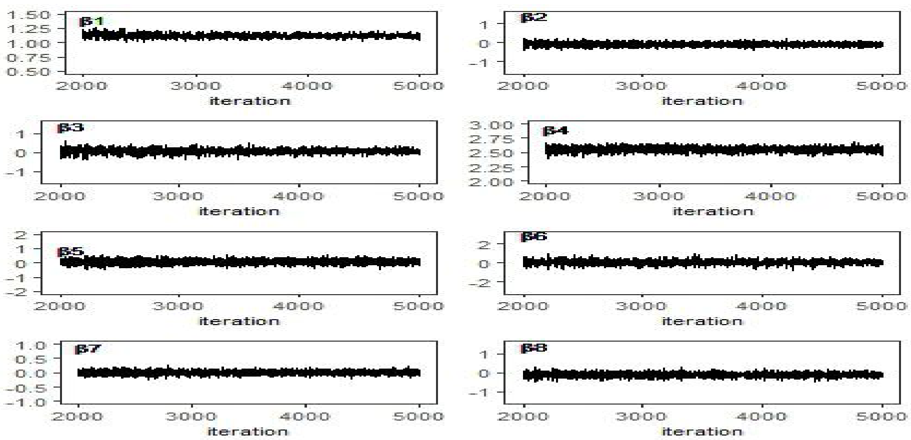

图3 误差为正态分布时，$T=10$，$N=30$，τ=0.9分位点AEP方法的参数双层迭代轨迹图

（2）误差为学生分布

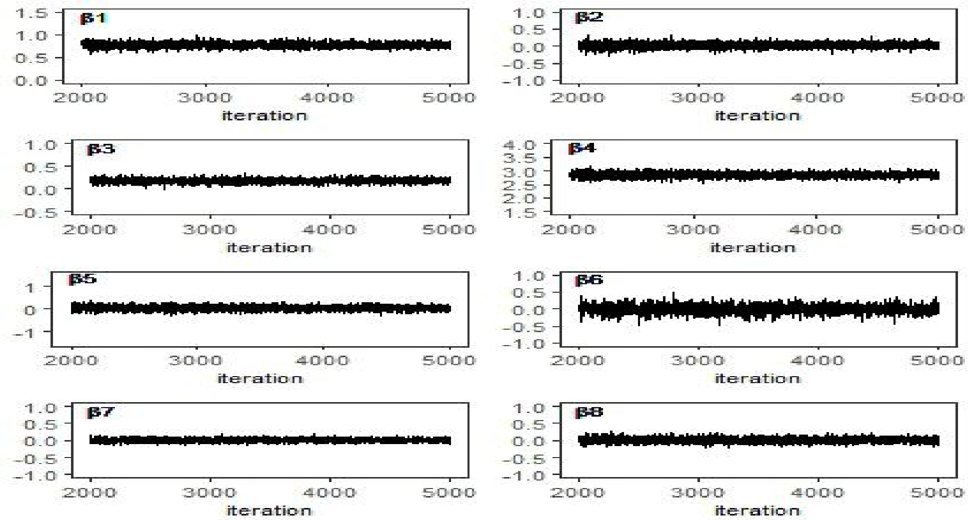

图4 误差为学生分布时，$T=10$， $N=30$， τ=0.1分位点AEP方法的参数双层迭代轨迹图

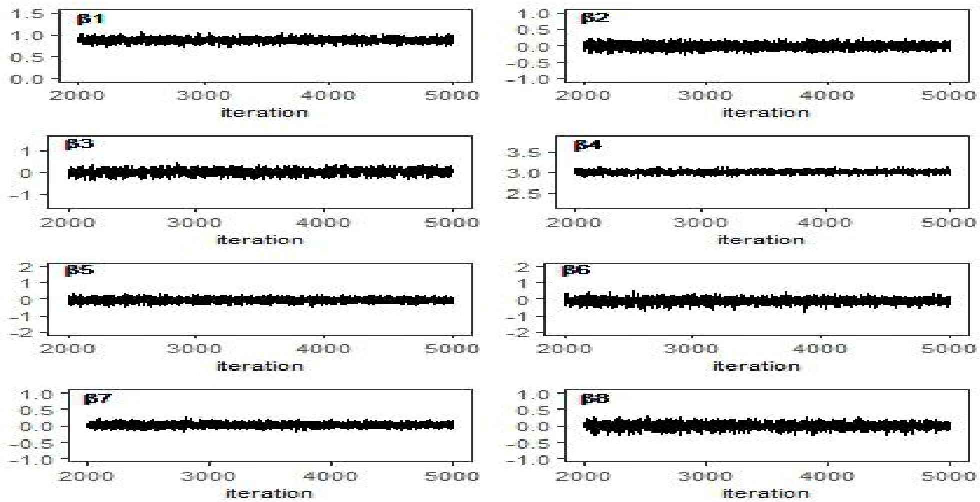

图5 误差为学生分布时，$T=10$， $N=30$， τ=0.5分位点AEP方法的参数双层迭代轨迹图

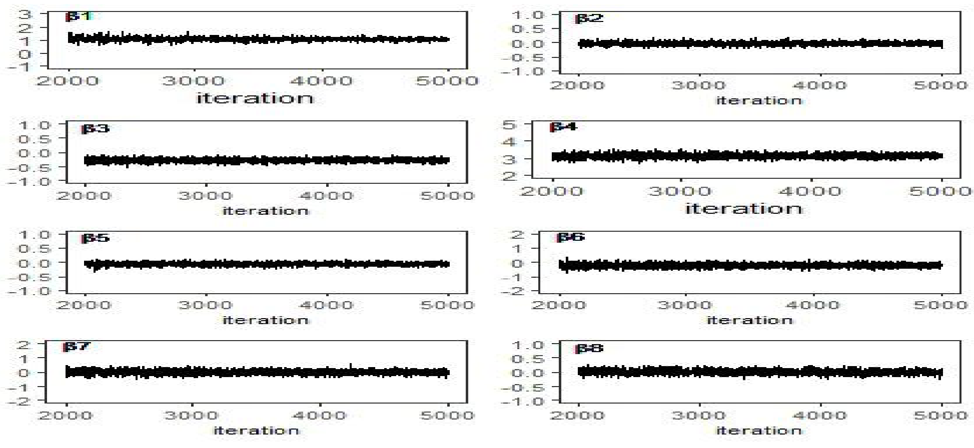

图6 误差为学生分布时，$T=10$， $N=30$， τ=0.9分位点AEP方法的参数双层迭代轨迹图

（3）误差为卡方分布

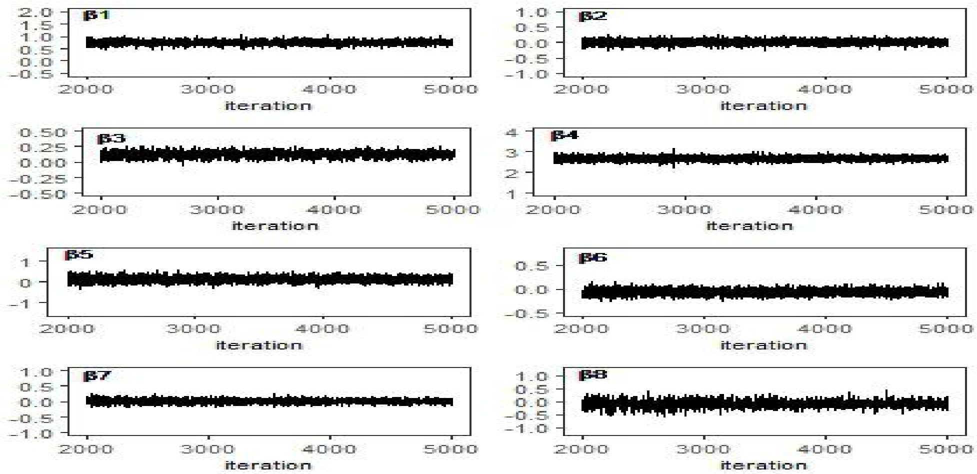

图7　误差为卡方分布时，$T=10$，$N=30$，τ=0.1分位点AEP方法的参数双层迭代轨迹图

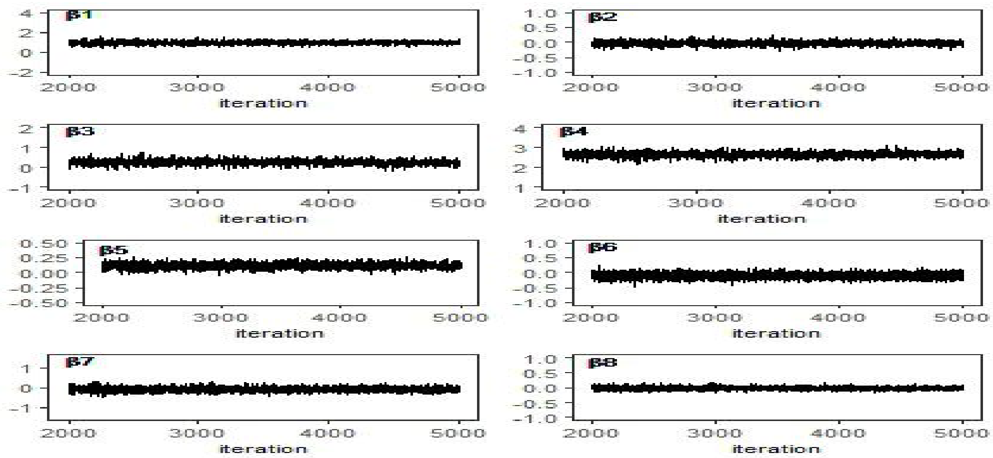

图8　误差为卡方分布时，$T=10$，$N=30$，τ=0.5分位点AEP方法的参数双层迭代轨迹图

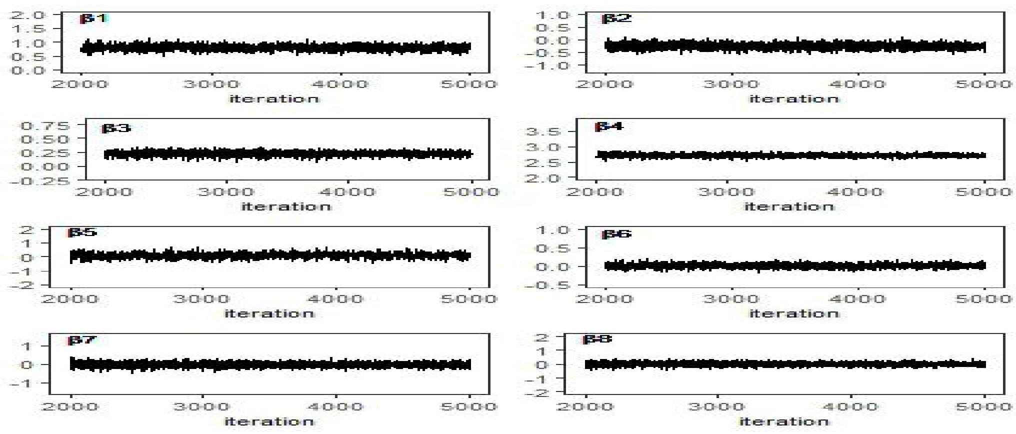

图9　误差为卡方分布时，$T=10$，$N=30$，τ=0.9分位点AEP方法的参数双层迭代轨迹图

2.样本容量为 $T=50$, $N=50$ 时，AEP 方法的参数双层迭代轨迹图。

（1）误差为正态分布

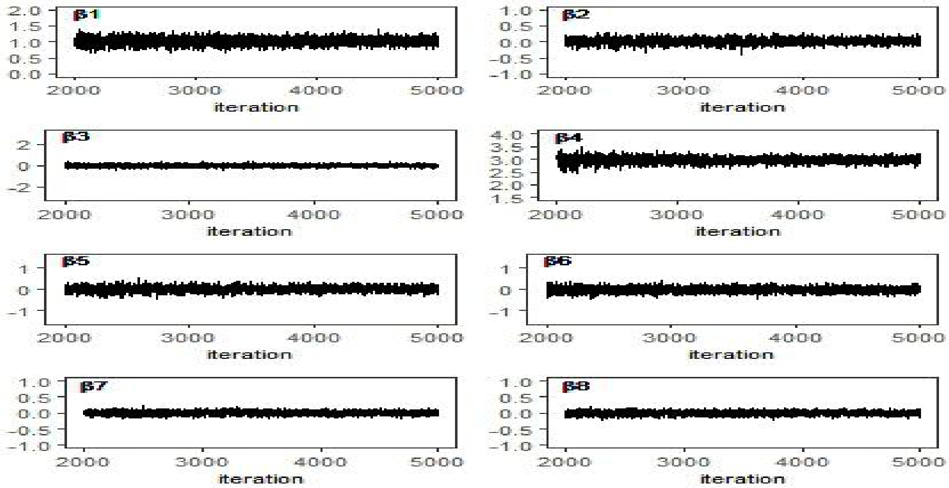

图 10　误差为正态分布时，$T=50$，$N=50$，τ=0.1 分位点 AEP 方法的参数双层迭代轨迹图

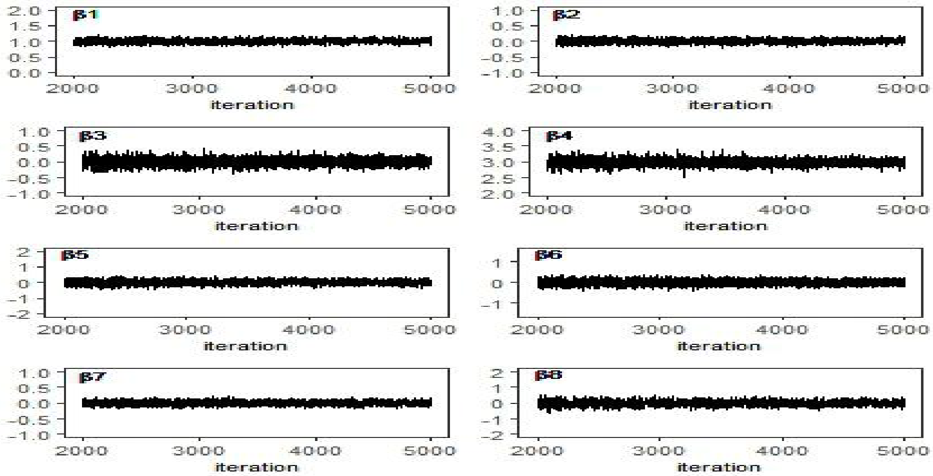

图 11　误差为正态分布时，$T=50$，$N=50$，τ=0.5 分位点 AEP 方法的参数双层迭代轨迹图

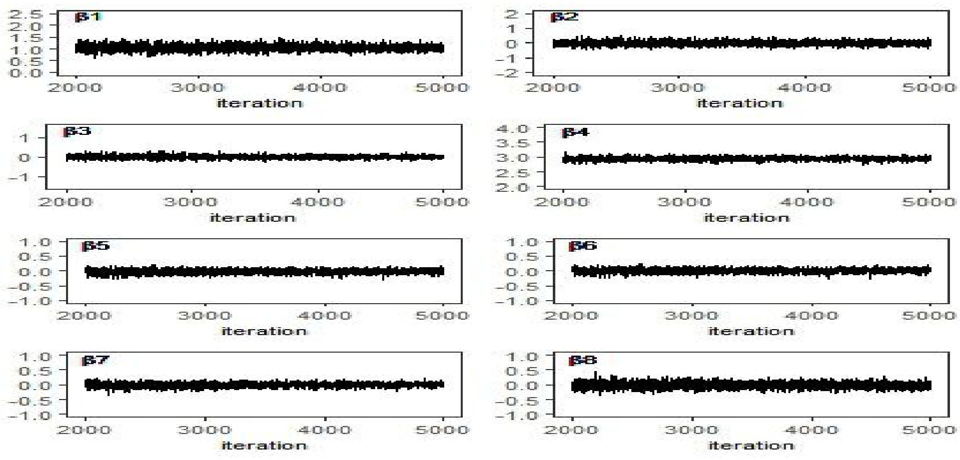

图 12 误差为正态分布时，$T=50$，$N=50$，τ=0.9 分位点 AEP 方法的参数双层迭代轨迹图

（2）误差为学生分布

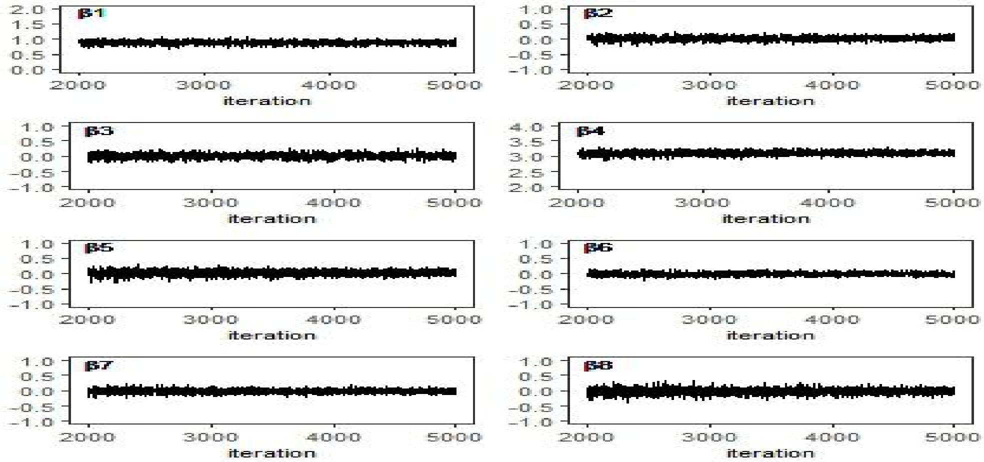

图13　误差为学生分布时，$T=50$，$N=50$，τ=0.1分位点AEP方法的参数双层迭代轨迹图

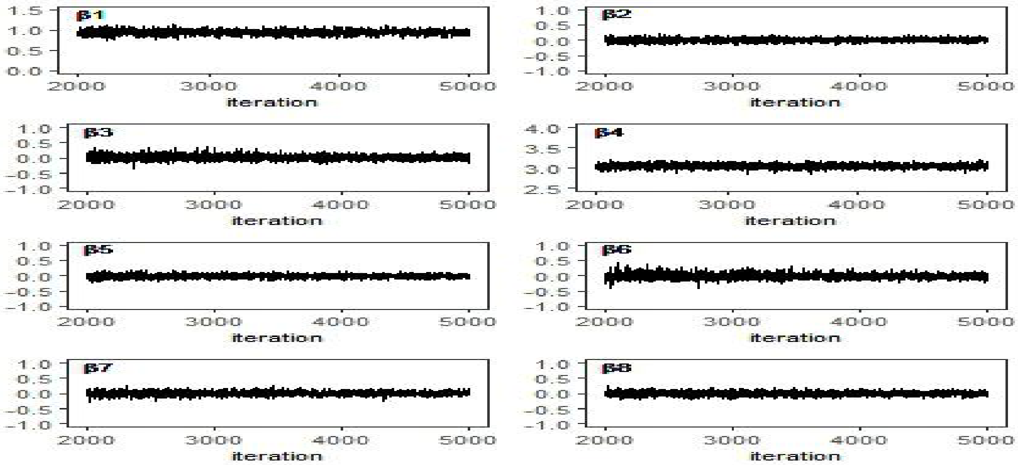

图14　误差为学生分布时，$T=50$，$N=50$，τ=0.5分位点AEP方法的参数双层迭代轨迹图

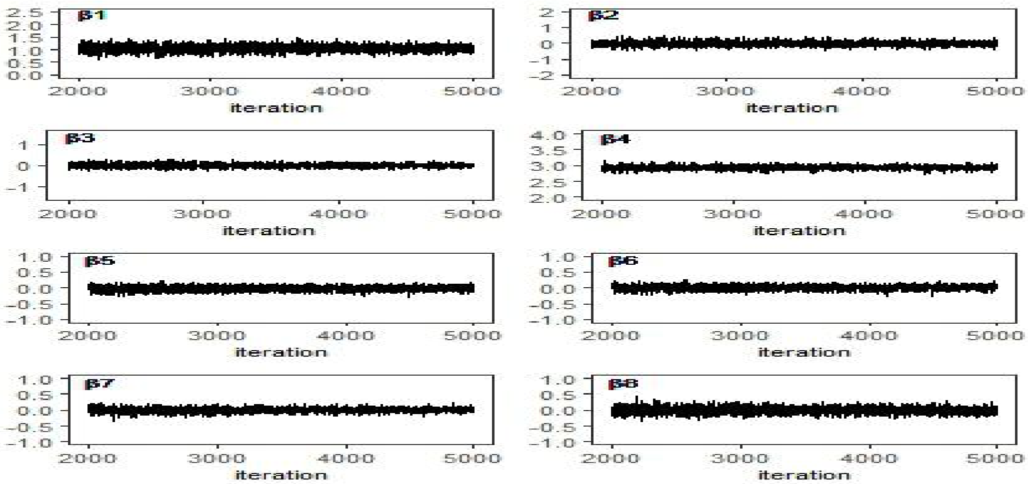

图15　误差为学生分布时，$T=50$，$N=50$，τ=0.9分位点AEP方法的参数双层迭代轨迹图

（3）误差为卡方分布

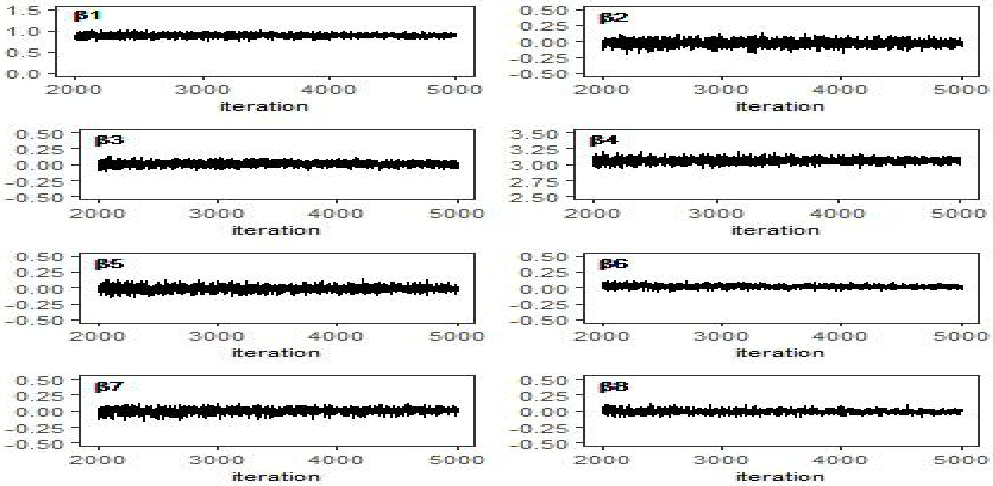

图16　误差为卡方分布时，$T=50$，$N=50$，τ=0.1分位点AEP方法的参数双层迭代轨迹图

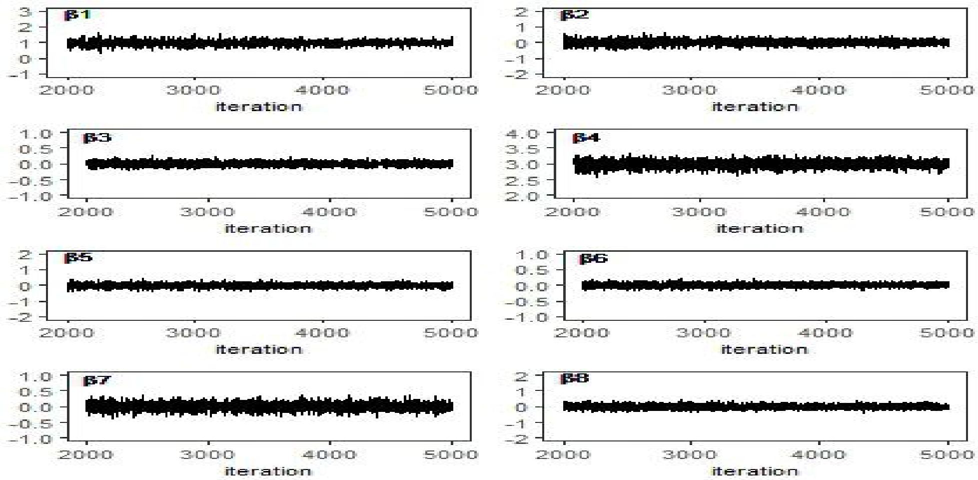

图17　误差为卡方分布时，$T=50$，$N=50$，τ=0.5分位点AEP方法的参数双层迭代轨迹图

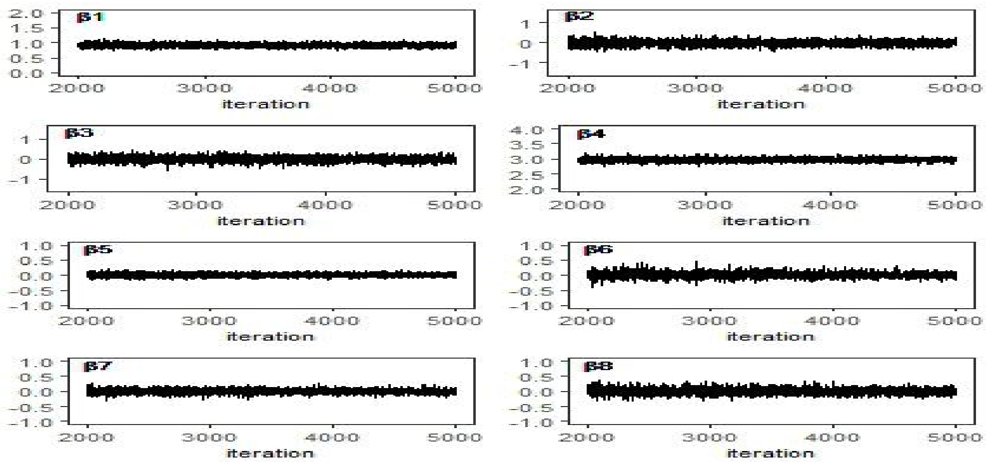

图18　误差为卡方分布时，$T=50$，$N=50$，τ=0.9分位点AEP方法的参数双层迭代轨迹图

附图 3

（三）实证研究中 AEP 方法的参数取 2 个不同初值的双层迭代轨迹图

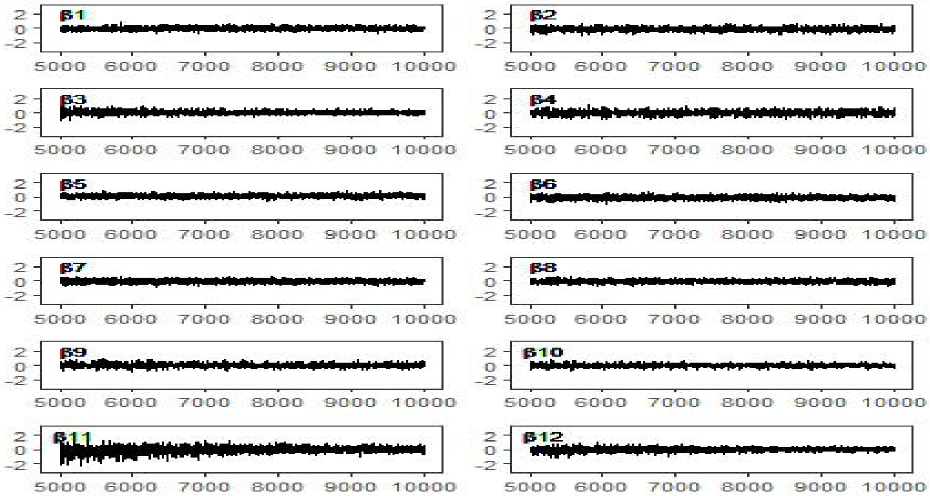

图 1 实证研究中，$\tau = 0.1$ 分位点 AEP 方法的参数双层迭代轨迹图

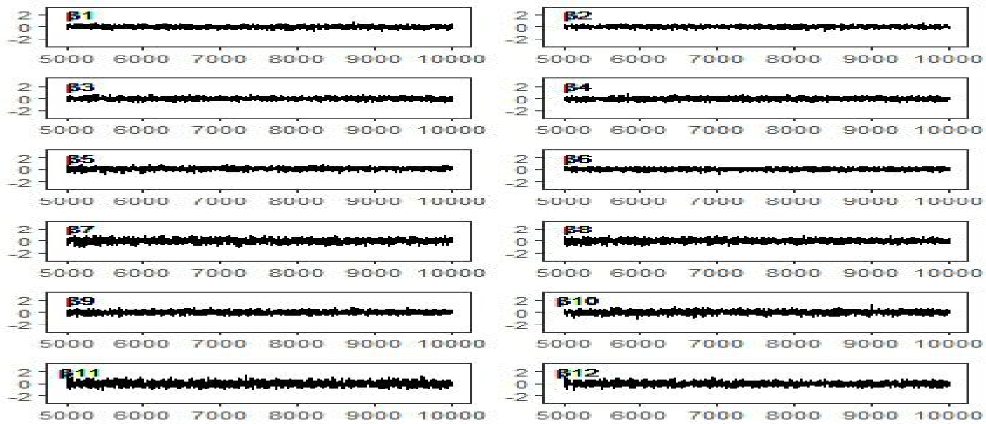

图 2 实证研究中，$\tau = 0.5$ 分位点 AEP 方法的参数双层迭代轨迹图

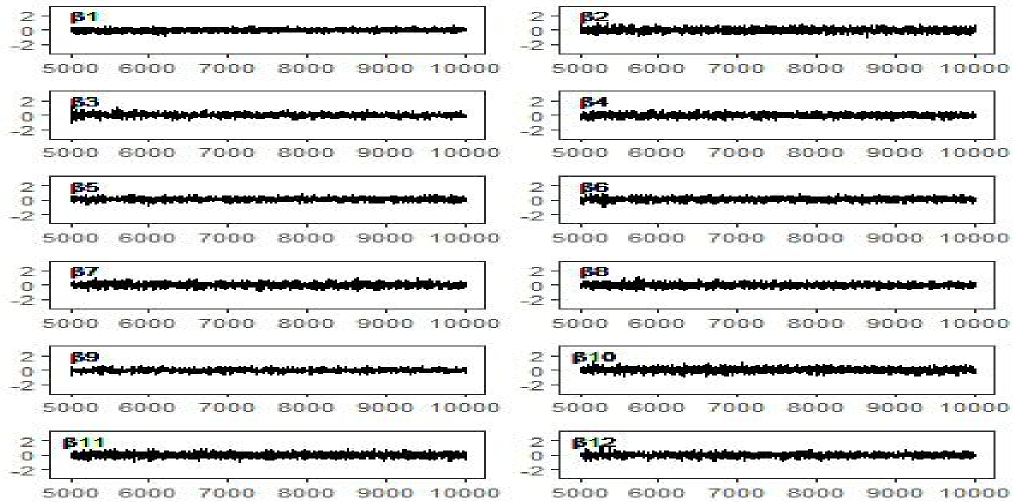

图 3 实证研究中，$\tau = 0.9$ 分位点 AEP 方法的参数双层迭代轨迹图